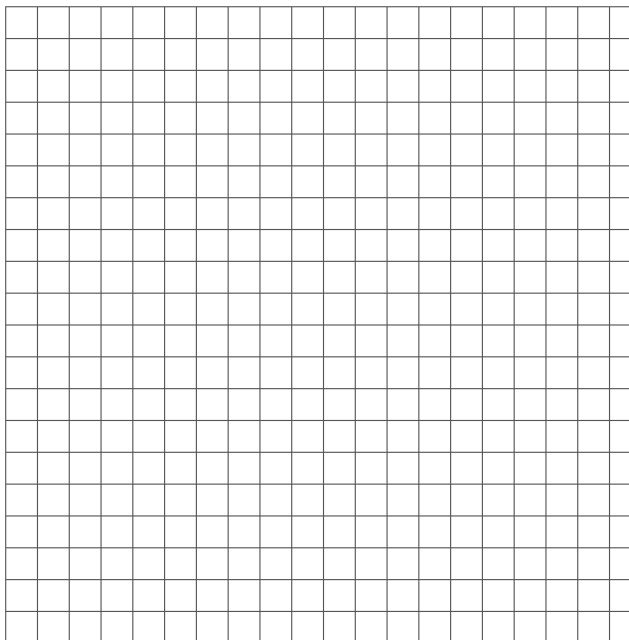# Logarithms and Logarithmic Functions

These notes are intended as an introduction to the topic of logarithms. They include material from section 5.4 (p. 375 – 380) in your workbook. You should also read the section for more complete explanations and additional examples.

**Logarithms**

Use the table of values below to graph of $f(x) = 2^x$ and its inverse.

| $x$ | $f(x)$ |
|-----|--------|
| −3 | |
| −2 | |
| −1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |

The inverse of the exponential function $y = a^x$, written $x = a^y$, is called a **logarithmic function**. The exponent $y$ is known as a **logarithm**.

Logarithmic functions are more commonly written in logarithmic form as

$$y = \log_a x \qquad \begin{cases} a > 0 \\ a \neq 1 \\ x > 0 \end{cases}$$

This is read as log base $a$ of $x$. $y$ is the logarithm, $a$ is the base, and $x$ is the argument.

**Example 1 (sidebar p. 377)**

a) Write each exponential expression as a logarithmic expression.

   i)   $3^3 = 27$

   ii)   $5^{-2} = \dfrac{1}{25}$

   iii)   $4^0 = 1$

b) Write each logarithmic expression as an exponential expression.

   i)   $\log_7 49 = 2$

   ii)   $\log_4 \left( \dfrac{1}{64} \right) = -3$

   iii)   $\log_{10} \left( \dfrac{1}{10000} \right) = -4$

**Common Logarithm**

A logarithmic function with base 10, $y = \log_{10} x$, is called a **common logarithmic function**. Any logarithm written without a base (e.g. $y = \log x$) is assumed to be base 10.

On your calculator, use the LOG key to evaluate a logarithm with base 10.

**Example (not in workbook)**
Evaluate $y = \log 100$.

If the argument is the unknown, you can use the $10^x$ key (2nd function, LOG) to determine the value of $x$.

**Example (not in workbook)**
Determine the value of $x$ in $\log x = 2$.

**Natural Logarithm**

A logarithmic function with base $e$, $y = \log_e x$, is called a **natural logarithmic function**. It is more commonly written as
$$y = \ln x$$

On your calculator, use the LN key to evaluate a logarithm with base $e$. Use the $e^x$ key (2nd function, LN) to solve for the argument.

**Example (not in workbook)**
Solve each of the following:

a)  $y = \ln 5$

b)  $\ln x = 12$

**Homework**: #5, 6, 8 in the exercises (p. 381 – 385). Answers on p. 386.